Crystal Bases

Another new title in the popular ULECT series; affordably-priced, softcover, advanced-level topics; presents a great entry point into a rapidly-developing topic area; self-contained and suitable for use as an independent study text.

Projective Representations of the Symmetric Groups

Combinatorial Problems Related to the Representation Theory of the Symmetric Group

Analytic combinatorics aims to enable precise quantitative predictions of the properties of large combinatorial structures. The theory has emerged over recent decades as essential both for the analysis of algorithms and for the study of scientific models in many disciplines, including probability theory, statistical physics, computational biology, and information theory. With a careful combination of symbolic enumeration methods and complex analysis, drawing heavily on generating functions, results of sweeping generality emerge that can be applied in particular to fundamental structures such as permutations, sequences, strings, walks, paths, trees, graphs and maps. This account is the definitive treatment of the topic. The authors give full coverage of the underlying mathematics and a thorough treatment of both classical and modern applications of the theory. The text is complemented with exercises, examples, appendices and notes to aid understanding. The book can be used for an advanced undergraduate or a graduate course, or for self-study.

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The representation theory of symmetric groups is a classical topic that, since the pioneering work of Frobenius, Schur and Young, has grown into a huge body of theory, with many important connections to other areas of mathematics and physics. This self-contained book provides a detailed introduction to the subject, covering classical topics such as the Littlewood–Richardson rule and the Schur–Weyl duality. Importantly, the authors also present many recent advances in the area, including Lassalle’s character formulas, the theory of partition algebras, and an exhaustive exposition of the approach developed by A. M. Vershik and A. Okounkov. A wealth of examples and exercises makes this an ideal textbook for graduate students. It will also serve as a useful reference for more experienced researchers across a range of areas, including algebra, computer science, statistical mechanics and theoretical physics.

The Symmetric Group
Representation Theory of Symmetric Groups is the most up-to-date abstract algebra book on the subject of symmetric groups and representation theory. Utilizing new research and results, this book can be studied from a combinatorial, algorithmic or algebraic viewpoint. This book is an excellent way of introducing today’s students to representation theory of the symmetric groups, namely classical theory. From there, the book explains how the theory can be extended to other related combinatorial algebras like the Iwahori-Hecke algebra. In a clear and concise manner, the author presents the case that most calculations on symmetric group can be performed by utilizing appropriate algebras of functions. Thus, the book explains how some Hopf algebras (symmetric functions and generalizations) can be used to encode most of the combinatorial properties of the representations of symmetric groups. Overall, the book is an innovative introduction to representation theory of symmetric groups for graduate students and researchers seeking new ways of thought.

Representation Theory
This graduate-level text provides a thorough grounding in the representation theory of finite groups over fields and rings. The book provides a balanced and comprehensive account of the subject, detailing the methods needed to analyze representations that arise in many areas of mathematics. Key topics include the construction and use of character tables, the role of induction and restriction, projective and simple modules for group algebras, indecomposable representations, Brauer characters, and block theory. This classroom-tested text provides motivation through a large number of examples and exercises.
Representation Theory of Finite Groups: a Guidebook

Very roughly speaking, representation theory studies symmetry in linear spaces. It is a beautiful mathematical subject which has many applications, ranging from number theory and combinatorics to geometry, probability theory, quantum mechanics, and quantum field theory. The goal of this book is to give a ``holistic'' introduction to representation theory, presenting it as a unified subject which studies representations of associative algebras and treating the representation theories of groups, Lie algebras, and quivers as special cases. Using this approach, the book covers a number of standard topics in the representation theories of these structures. Theoretical material in the book is supplemented by many problems and exercises which touch upon a lot of additional topics; the more difficult exercises are provided with hints. The book is designed as a textbook for advanced undergraduate and beginning graduate students. It should be accessible to students with a strong background in linear algebra and a basic knowledge of abstract algebra.

Lectures in Algebraic Combinatorics

This text systematically presents the basics of quantum mechanics, emphasizing the role of Lie groups, Lie algebras, and their unitary representations. The mathematical structure of the subject is brought to the fore, intentionally avoiding significant overlap with material from standard physics courses in quantum mechanics and quantum field theory. The level of presentation is attractive to mathematics students looking to learn about both quantum mechanics and representation theory, while also appealing to physics students who would like to know more about the mathematics underlying the subject. This text showcases the numerous differences between typical mathematical and physical treatments of the subject. The latter portions of the book focus on central mathematical objects that occur in the Standard Model of particle physics, underlining the deep and intimate connections between mathematics and the physical world. While an elementary physics course of some kind would be helpful to the reader, no specific background in physics is assumed, making this book accessible to students with a grounding in multivariable calculus and linear algebra. Many exercises are provided to develop the reader's understanding of and facility in quantum-theoretical concepts and calculations.

Unitary Symmetry And Combinatorics

This volume, devoted to the 70th birthday of the well-known St. Petersburg mathematician A. M. Vershik, contains a collection of articles by participants in the conference "Representation Theory, Dynamical Systems, and Asymptotic Combinatorics", held in St. Petersburg in June of 2004. The book is suitable for graduate students and researchers interested in combinatorial and dynamical aspects of group representation theory.

Young Tableaux

Includes a rich variety of exercises to accompany the exposition of Coxeter groups. Coxeter groups have already been exposited from algebraic and
The Symmetric Group

One of the most classical areas of algebra, the theory of symmetric functions and orthogonal polynomials has long been known to be connected to combinatorics, representation theory, and other branches of mathematics. Written by perhaps the most famous author on the topic, this volume explains some of the current developments regarding these connections. It is based on lectures presented by the author at Rutgers University. Specifically, he gives recent results on orthogonal polynomials associated with affine Hecke algebras, surveying the proofs of certain famous combinatorial conjectures.

Representations and Characters of Groups

This book is a gentle introduction to the enumerative part of combinatorics suitable for study at the advanced undergraduate or beginning graduate level. In addition to covering all the standard techniques for counting combinatorial objects, the text contains material from the research literature which has never before appeared in print, such as the use of quotient posets to study the Möbius function and characteristic polynomial of a partially ordered set, or the connection between quasisymmetric functions and pattern avoidance. The book assumes minimal background, and a first course in abstract algebra should suffice. The exposition is very reader friendly: keeping a moderate pace, using lots of examples, emphasizing recurring themes, and frankly expressing the delight the author takes in mathematics in general and combinatorics in particular.

The Representation Theory of the Symmetric Groups

Combinatorics: The Art of Counting

Proofs and Confirmations

Written by one of the foremost experts in the field, Algebraic Combinatorics is a unique undergraduate textbook that will prepare the next generation of pure and applied mathematicians. The combination of the author's extensive knowledge of combinatorics and classical and practical tools from algebra will inspire motivated students to delve deeply into the fascinating interplay between algebra and combinatorics. Readers will be able to apply their newfound knowledge to mathematical, engineering, and business models. The text is primarily intended for use in a one-semester advanced undergraduate course in algebraic combinatorics, enumerative combinatorics, or graph theory. Prerequisites include a basic knowledge of linear algebra over a field, existence of finite fields, and group theory. The topics in each chapter build on one another and include extensive problem sets as well as hints to selected exercises. Key topics include walks on graphs, cubes and the Radon transform, the Matrix–Tree Theorem, and the Sperner property. There are also three appendices on purely enumerative aspects of combinatorics related to the chapter material: the RSK algorithm, plane partitions, and the enumeration of labeled trees. Richard Stanley is currently professor of Applied Mathematics at the Massachusetts Institute of Technology.
The book contains detailed descriptions of the many exciting recent developments in the combinatorics of the space of diagonal harmonics, a topic at the forefront of current research in algebraic combinatorics. These developments led in turn to some surprising discoveries in the combinatorics of Macdonald polynomials, which are described in Appendix A. The book is appropriate as a text for a topics course in algebraic combinatorics, a volume for self-study, or a reference text for researchers in any area which involves symmetric functions or lattice path combinatorics. The book contains expository discussions of some topics in the theory of symmetric functions, such as the practical uses of plethystic substitutions, which are not treated in depth in other texts. Exercises are interspersed throughout the text in strategic locations, with full solutions given in Appendix C.

Read Free The Symmetric Group Representations Combinatorial Algorithms And Symmetric Functions Graduate Texts In Mathematics

The Representation Theory of the Symmetric Group

This book is the first of its kind covering the topic. It offers a substantially simplified treatment of the original proofs. The book is a solid reference source for experts. It will also serve as a good introduction to students and beginning researchers since each chapter contains exercises and there is an appendix containing a quick development of the representation theory of algebras.

Symmetric Functions and Orthogonal Polynomials

Quantum Theory, Groups and Representations

This monograph integrates unitary symmetry and combinatorics, showing in great detail how the coupling of angular momenta in quantum mechanics is related to binary trees, trivalent trees, cubic graphs, MacMahon's master theorem, and other basic combinatorial concepts. A comprehensive theory of recoupling matrices for quantum angular momentum is developed. For the general unitary group, polynomial forms in many variables called matrix Schur functions have the remarkable property of giving all irreducible representations of the general unitary group and are the basic objects of study. The structure of these irreducible polynomials and the reduction of their Kronecker product is developed in detail, as is the theory of tensor operators.

Flag Varieties

This unique book provides the first introduction to crystal base theory from the combinatorial point of view. Crystal base theory was developed by Kashiwara and Lusztig from the perspective of quantum groups. Its power comes from the fact that it addresses many questions in representation theory and mathematical physics by combinatorial means. This book approaches the subject directly from combinatorics, building crystals through
Local axioms (based on ideas by Stembridge) and virtual crystals. It also emphasizes parallels between the representation theory of the symmetric and general linear groups and phenomena in combinatorics. The combinatorial approach is linked to representation theory through the analysis of Demazure crystals. The relationship of crystals to tropical geometry is also explained.

Contents:
- Introduction
- Kashiwara Crystals
- Crystals of Tableaux
- Stembridge Crystals
- Virtual, Fundamental, and Normal Crystals
- Crystals of Tableaux II
- Insertion Algorithms
- The Plactic Monoid
- Bicrystals and the Littlewood–Richardson Rule
- Crystals for Stanley Symmetric Functions
- Patterns and the Weyl Group Action
- The $\beta_\infty$ Crystal
- Demazure Crystals
- The $\star$-Involution of $\beta_\infty$ Crystals and Tropical Geometry
- Further Topics

Readership: Graduate students and researchers interested in understanding from a viewpoint of combinatorics on crystal base theory.

Representation Theory of Finite Groups

This book is intended to present group representation theory at a level accessible to mature undergraduate students and beginning graduate students. This is achieved by mainly keeping the required background to the level of undergraduate linear algebra, group theory and very basic ring theory. Module theory and Wedderburn theory, as well as tensor products, are deliberately avoided. Instead, we take an approach based on discrete Fourier Analysis. Applications to the spectral theory of graphs are given to help the student appreciate the usefulness of the subject. A number of exercises are included. This book is intended for a 3rd/4th undergraduate course or an introductory graduate course on group representation theory. However, it can also be used as a reference for workers in all areas of mathematics and statistics.

The Symmetric Group

Bijective proofs are some of the most elegant and powerful techniques in all of mathematics. Suitable for readers without prior background in algebra or combinatorics, Bijective Combinatorics presents a general introduction to enumerative and algebraic combinatorics that emphasizes bijective methods. The text systematically develops the mathematical tools, such as basic counting rules, recursions, inclusion-exclusion techniques, generating functions, bijective proofs, and linear-algebraic methods, needed to solve enumeration problems. These tools are used to analyze many combinatorial structures, including words, permutations, subsets, functions, compositions, integer partitions, graphs, trees, lattice paths, multisets, rook placements, set partitions, Eulerian tours, derangements, posets, tilings, and abaci. The book also delves into algebraic aspects of combinatorics, offering detailed treatments of formal power series, symmetric groups, group actions, symmetric polynomials, determinants, and the combinatorial calculus of tableaux. Each chapter includes summaries and extensive problem sets that review and reinforce the material. Lucid, engaging, yet fully rigorous, this text describes a host of combinatorial techniques to help solve complicated enumeration problems. It covers the basic principles of enumeration, giving due attention to the role of bijective proofs in enumeration theory.

Representation Theory

This is the first comprehensive text in this important and active area of research.

Iwahori-Hecke Algebras and Schur Algebras of the Symmetric Group
This book provides a modern introduction to the representation theory of finite groups. Now in its second edition, the authors have revised the text and added much new material. The theory is developed in terms of modules, since this is appropriate for more advanced work, but considerable emphasis is placed upon constructing characters. Included here are the character tables of all groups of order less than 32, and all simple groups of order less than 1000. Applications covered include Burnside's pabq theorem, the use of character theory in studying subgroup structure and permutation groups, and how to use representation theory to investigate molecular vibration. Each chapter features a variety of exercises, with full solutions provided at the end of the book. This will be ideal as a course text in representation theory, and in view of the applications, will be of interest to chemists and physicists as well as mathematicians.

Introduction to Representation Theory

"It is known that the set of p-regular partitions of n, the set of \( \Gamma_0 \) partitions of n, and the set of Alperin weights of the symmetric group \( S_n \) all have equal size. We investigate the relationship between partitions and representations of the symmetric group in search of a natural bijection between the Alperin weights and the partitions in \( \Gamma_0 \). Through this search, we were able to classify the abacus display of one-ladder partitions and also describe circumstances when the set of \( \Gamma_0 \) partitions of n is in face equal to the set of p-regular partitions of n."--Abstract.

Representation Theory of Symmetric Groups

This book brings together many of the important results in this field. From the reviews: "A classic gets even better. The edition has new material including the Novelli-Pak-Stoyanovskii bijective proof of the hook formula, Stanley's proof of the sum of squares formula using differential posets, Fomin's bijective proof of the sum of squares formula, group acting on posets and their use in proving unimodality, and chromatic symmetric functions." --ZENTRALBLATT MATH

Introduction to Quantum Groups and Crystal Bases

Representation Theory of Symmetric Groups is the most up-to-date abstract algebra book on the subject of symmetric groups and representation theory. Utilizing new research and results, this book can be studied from a combinatorial, algorithmic or algebraic viewpoint. This book is an excellent way of introducing today's students to representation theory of the symmetric groups, namely classical theory. From there, the book explains how the theory can be extended to other related combinatorial algebras like the Iwahori-Hecke algebra. In a clear and concise manner, the author presents the case that most calculations on symmetric group can be performed by utilizing appropriate algebras of functions. Thus, the book explains how some Hopf algebras (symmetric functions and generalizations) can be used to encode most of the combinatorial properties of the representations of symmetric groups. Overall, the book is an innovative introduction to representation theory of symmetric groups for graduate students and researchers seeking new ways of thought.

Combinatorics of Coxeter Groups
Representation Theory, Dynamical Systems, and Asymptotic Combinatorics

This is an introduction to recent developments in algebraic combinatorics and an illustration of how research in mathematics actually progresses. The author recounts the story of the search for and discovery of a proof of a formula conjectured in the late 1970s: the number of n x n alternating sign matrices, objects that generalize permutation matrices. While apparent that the conjecture must be true, the proof was elusive. Researchers became drawn to this problem, making connections to aspects of invariant theory, to symmetric functions, to hypergeometric and basic hypergeometric series, and, finally, to the six-vertex model of statistical mechanics. All these threads are brought together in Zeilberger's 1996 proof of the original conjecture. The book is accessible to anyone with a knowledge of linear algebra. Students will learn what mathematicians actually do in an interesting and new area of mathematics, and even researchers in combinatorics will find something new here.
The primary goal of these lectures is to introduce a beginner to the finite dimensional representations of Lie groups and Lie algebras. Since this goal is shared by quite a few other books, we should explain in this Preface how our approach differs, although the potential reader can probably see this better by a quick browse through the book. Representation theory is simple to define: it is the study of the ways in which a given group may act on vector spaces. It is almost certainly unique, however, among such clearly delineated subjects, in the breadth of its interest to mathematicians. This is not surprising: group actions are ubiquitous in 20th century mathematics, and where the object on which a group acts is not a vector space, we have learned to replace it by one that is—e.g., a cohomology group, tangent space, etc. As a consequence, many mathematicians other than specialists in the field—or even those who think they might want to be—come in contact with the subject in various ways. It is for such people that this text is designed. To put it another way, we intend this as a book for beginners to learn from and not as a reference. This idea essentially determines the choice of material covered here. As simple as is the definition of representation theory given above, it fragments considerably when we try to get more specific.
Read Free The Symmetric Group Representations Combinatorial Algorithms And Symmetric Functions Graduate Texts In Mathematics

This book discusses the representation theory of symmetric groups, the theory of symmetric functions and the polynomial representation theory of general linear groups. The first chapter provides a detailed account of necessary representation-theoretic background. An important highlight of this book is an innovative treatment of the Robinson–Schensted–Knuth correspondence and its dual by extending Viennot's geometric ideas. Another unique feature is an exposition of the relationship between these correspondences, the representation theory of symmetric groups and alternating groups and the theory of symmetric functions. Schur algebras are introduced very naturally as algebras of distributions on general linear groups. The treatment of Schur–Weyl duality reveals the directness and simplicity of Schur's original treatment of the subject. In addition, each exercise is assigned a difficulty level to test readers' learning. Solutions and hints to most of the exercises are provided at the end.

Representation Theory of Symmetric Groups

The notion of a "quantum group" was introduced by V.G. Dinfeld and M. Jimbo, independently, in their study of the quantum Yang-Baxter equation arising from 2-dimensional solvable lattice models. Quantum groups are certain families of Hopf algebras that are deformations of universal enveloping algebras of Kac-Moody algebras. And over the past 20 years, they have turned out to be the fundamental algebraic structure behind many branches of mathematics and mathematical physics, such as solvable lattice models in statistical mechanics, topological invariant theory of links and knots, representation theory of Kac-Moody algebras, representation theory of algebraic structures, topological quantum field theory, geometric representation theory, and C*-algebras. In particular, the theory of "crystal bases" or "canonical bases" developed independently by M. Kashiwara and G. Lusztig provides a powerful combinatorial and geometric tool to study the representations of quantum groups. The purpose of this book is to provide an elementary introduction to the theory of quantum groups and crystal bases, focusing on the combinatorial aspects of the theory.